$V/W_1 = \{a + W_1 : a \in V\}$ = QUOTIENT sp. rel. to W_1

$V/W_2 = \{a + W_2 : a \in V\}$ = QUOTIENT sp. relat. to W_2.

Basis of $V/W_1 = \{(0,1,0) + W_1, (0,0,1) + W_1\}$

Basis of $V/W_2 = \{(1,0,0) + W_2\}$

(17) COUNTING OF BASIS AND SUBSPACE

Let $V(F)$ be a f.d. V.S say $\dim(V) = n$ an $|F| = p$. Then,

① number of elts in $V(F) = |F|^{\dim V} = p^n$

② Number of ordered basis of $V = (p^n - 1)(p^n - p) \ldots (p^n - p^{n-1})$

③ Number of unordered basis of V

$$= \frac{(p^n - 1)(p^n - p) \ldots (p^n - p^{n-1})}{n!}$$

④ Total no. of subspaces of dim. r of V

$$= \frac{(p^n - 1)(p^n - p)(p^n - p^2) \ldots (p^n - p^{n-1})}{(p^r - 1)(p^r - p)(p^r - p^2) \ldots (p^r - p^{r-1})}$$

Q: If $\dim V(F) = 4$ and $|F| = 2$. Then,

(a) $|V| = 2^4 = 16$

(b) no. of ordered basis =

(c) Total no. of subspaces of dim. 3 is

$$= \frac{(2^4 - 1)(2^4 - 2)(2^4 - 2^2)(2^4 - 2^3)}{(2^3 - 1)(2^3 - 2)(2^3 - 2^2)} = \frac{15 \times 14 \times 12 \times 8}{7 \times 6 \times 4} = 15$$

$V/W_1 = \{a + W_1 : a \in V\}$ = QUOTIENT sp. rel. to W_1

$V/W_2 = \{a + W_2 : a \in V\}$ = QUOTIENT sp. relate to W_2.

Basis of $V/W_1 = \{(0,1,0) + W_1, (0,0,1) + W_1\}$

Basis of $V/W_2 = \{(1,0,0) + W_2\}$

(17) COUNTING OF BASIS AND SUBSPACE

Let $V(F)$ be a f.d. V.S say $\dim(V) = n$ and $|F| = p$. Then,

(1) number of elts in $V(F) = |F|^{\dim V} = p^n$

(2) Number of ordered basis of $V = (p^n - 1)(p^n - p) \cdots (p^n - p^{n-1})$

(3) Number of unordered basis of V

$$= \frac{(p^n-1)(p^n-p)\cdots(p^n-p^{n-1})}{n!}$$

(4) Total no. of subspaces of dim. r of V

$$= \frac{(p^n-1)(p^n-p)(p^n-p^2)\cdots(p^n-p^{n-1})}{(p^r-1)(p^r-p)(p^r-p^2)\cdots(p^r-p^{r-1})}$$

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